Exercise 65

At what points on the curve $y = \sin x + \cos x$, $0 \le x \le 2\pi$, is the tangent line horizontal?

Solution

Start by calculating the derivative.

$$\begin{aligned} y' &= \frac{d}{dx}(\sin x + \cos x) \\ &= \frac{d}{dx}(\sin x) + \frac{d}{dx}(\cos x) \\ &= (\cos x) + (-\sin x) \\ &= \cos x - \sin x \end{aligned}$$

Set the result equal to zero.

$$\cos x - \sin x = 0$$

$$\cos x = \sin x$$

The values of x between 0 and 2π that satisfy this equation are

$$x = \frac{\pi}{4} \quad \text{and} \quad x = \frac{5\pi}{4}.$$

Determine the corresponding y-coordinates by plugging these values of x back into the given function.

$$x = \frac{\pi}{4}: \quad y = \sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$$

$$x = \frac{5\pi}{4}: \quad y = \sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = -\sqrt{2}$$

Therefore, the points on the curve that have horizontal tangent lines are

$$\left(\frac{\pi}{4}, \sqrt{2}\right) \quad \text{and} \quad \left(\frac{5\pi}{4}, -\sqrt{2}\right).$$

Below is a graph of the curve and these horizontal tangent lines over $0 \le x \le 2\pi$.